Planar and Surface Graphical Models which are Easy

Vladimir Y. Chernyak[a,b] and Michael Chertkov[b]

[a,b]*Department of Chemistry, Wayne State University, 5101 Cass Ave,Detroit, MI 48202*
[b]*Center for Nonlinear Studies and Theoretical Division, LANL, Los Alamos, NM 87545*

We describe a rich family of binary variables statistical mechanics models on planar graphs which are equivalent to Gaussian Grassmann Graphical models (free fermions). Calculation of partition function (weighted counting) in the models is easy (of polynomial complexity) as reduced to evaluation of determinants of matrixes linear in the number of variables. In particular, this family of models covers Holographic Algorithms of Valiant [1–3] and extends on the Gauge Transformations discussed in our previous works [4–8]. The planar part is based on [9].

We further extend our approach to the general case of surface graphs and demonstrate that, similar to the case of the dimer model considered in [10, 11], the partition function is given by an alternating sum of 2^{2g} determinants that correspond to 2^{2g} spinor structures on the embedding Riemann surface of genus g. This is achieved by considering the $\mathbb{Z}_2$-self-intersection invariant of immersions, and relating the spinor structures to the equivalence classes of Kasteleyn orientations on the so-called extended graph, associated with the original graph.

PACS numbers: 02.50.Tt, 64.60.Cn, 05.50.+q

[1] L. G. Valiant, SIAM Journal of Computing **31**, 1229 (2002).
[2] L. Valiant, Foundations of computer Science, 2004. Proceedings. 45th Annual IEEE Symposium on pp. 306-315 (2004).
[3] L. G. Valiant, SIAM Journal of Computing **37**, 1565 (2008).
[4] M. Chertkov and V. Chernyak, Phys. Rev. E **73**, 065102 (2006).
[5] M. Chertkov and V. Y. Chernyak, J. Stat. Mech. P06009 (2006).
[6] M. Chertkov, V. Y. Chernyak, and R. Teodorescu, J. Stat. Mech. P05003 (2008).
[7] V. Y. Chernyak and M. Chertkov, J. Stat. Mech. P12011 (2008).
[8] V. Y. Chernyak and M. Chertkov, J. Stat. Mech. P12012 (2008).
[9] V. Y. Chernyak and M. Chertkov, arXiv:0902.0320v1 (2009).
[10] D. Cimazoni and N. Reshetikhin, Commun. Math. Phys. **275**, 187 (2007).
[11] D. Cimazoni and N. Reshetikhin, Commun. Math. Phys. **281**, 445 (2008).